AQT Problem class 2

2C

$$I = \int e^{-\frac{1}{2}\underline{x}^T A \underline{x} - \varepsilon(x_1^4 + x_2^4)} \, d^n x$$

$$= c \int c^{-1} e^{-\frac{1}{2}\underline{x}^T A \underline{x}} \, e^{-\varepsilon(x_1^4 + x_2^4)} \, d^4 x$$

$$= c \left\langle e^{-\varepsilon(x_1^4 + x_2^4)} \right\rangle$$

where $c = \frac{(2\pi)^{n/2}}{\sqrt{\det(A)}}$

$$\left\langle e^{-\varepsilon(x_1^4 + x_2^4)} \right\rangle$$

$$= \left\langle \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\varepsilon(x_1^4 + x_2^4)\right)^n \right\rangle$$

$$= \left\langle 1 - \varepsilon(x_1^4 + x_2^4) + \frac{\varepsilon^2}{2}(x_1^4 + x_2^4)^2 \right\rangle$$

<u>linear in ε</u>

$$\langle x_k^4 \rangle = \langle x_k x_k x_k x_k \rangle + \langle x_k x_k x_k x_k \rangle + \langle x_k x_k x_k x_k \rangle$$

$$= 3 \left(A^{-1}\right)_{kk}^2$$

<u>quadratic in ε</u>

$$\langle x_k^8 \rangle = \langle x_k x_k x_k x_k x_k x_k x_k x_k \rangle + \dots$$

$$= \underbrace{7!!}_{=105} \left[\left(A^{-1}\right)_{kk} \right]^4$$

$\langle x_1^4 x_2^4 \rangle = \langle x_1 x_1 x_1 x_1 \; x_2 x_2 x_2 x_2 \rangle$

(a) contract only x with same index

$3 \, (A^{-1})_{11}^2 \quad 3 \, (A^{-1})_{22}^2$

(b) two contractions between x_1 & x_2

$72 (A^{-1})_{12}^2 \, (A^{-1})_{11} \, (A^{-1})_{22}$

multiplicity: $\binom{4}{2} = 6$ ways of dividing x_1 into two groups of two factors

same for x_2

2 ways of contracting x_1 & x_2

(c) all contractions between x_1 & x_2

$24 \quad (A^{-1})_{12}^4$

multiplicity: 4 ways of choosing x_2 contracted with first x_1, 3 for second, etc.

get $24 = 4!$

check: $9 + 72 + 24 = 105$

result:

$$
\begin{aligned}
I = \frac{2\pi}{\sqrt{\det A}} \Big[1 &- \varepsilon \left(3 (A^{-1})_{11} + 3 (A^{-1})_{22} \right) \\
&+ \frac{\varepsilon^2}{2} \Big(105 \, (A^{-1})_{11}^4 + 105 \, (A^{-1})_{11}^4 \\
&\quad + 2 \cdot 9 \, (A^{-1})_{11}^2 (A^{-1})_{22}^2 \\
&\quad + 2 \cdot 72 \, (A^{-1})_{11} (A^{-1})_{22} (A^{-1})_{12}^2 \\
&\quad + 2 \cdot 24 \, (A^{-1})_{12}^2 \Big) \Big]
\end{aligned}
$$

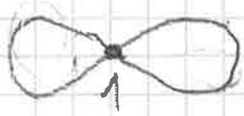
Feynman diagrams

$\overbracket{x_i \quad x_j}$

$\overbracket{x_i \quad x_i}$

e.g.

$\langle x_1^4 \rangle$ 3



$\langle x_1^4 x_2^4 \rangle$ for case (b) 72