Advanced Quantum Theory Revision Class

Problem based on 2015 exam, parts of question 3

Use perturbation theory to evaluate

$$\left\langle x(t_1)x(t_2)x(t_3)\exp\left(-\frac{i\epsilon}{\hbar}\int_0^t x(t')^3 dt'\right)\right\rangle$$

taking into account terms up to linear order in ϵ. Here the average $\langle \dots \rangle$ is defined as in the lectures about the anharmonic oscillator. Your result should involve integrals over products of factors $iG(t', t'')$ as defined in the course, and you don't need to evaluate these integrals explicitly. Also draw the corresponding Feynman diagrams.

2015 exam, question 2(a)i

Consider a bosonic (and spinless) quantum system with two single-particle states and the Hamiltonian

$$\hat{H} = -a_1^\dagger a_2 - a_2^\dagger a_1 + \frac{U}{2}(\hat{n}_1(\hat{n}_1 - 1) - \hat{n}_2(\hat{n}_2 - 1))$$

where U is a real number and we have $\hat{n}_i = a_i^\dagger a_i$. Here a_i is the annihilation operator for particles in the state i and $a_i^\dagger$ is the corresponding creation operator. In occupation number representation these operators are defined by

$$\begin{aligned} a_i^\dagger|\dots, n_i, \dots\rangle &= \sqrt{n_i + 1}|\dots, n_i + 1, \dots\rangle \\ a_i|\dots, n_i, \dots\rangle &= \sqrt{n_i}|\dots, n_i - 1, \dots\rangle. \end{aligned}$$

Evaluate $\hat{H}|n_1, n_2\rangle$. Using this result determine all energy levels for the case that two particles are present in the system. Also determine a matrix H_N such that all energy levels possible in the case of N particles (where N is an arbitrary integer) are eigenvalues of H_N.

General Remarks

The exam will have 2 questions without choice between them. It will be a closed book exam (you can't bring any notes) but formulas may be given on the cover sheet of the exam.

Scans from the revision class will be uploaded on the course webpage. The exams from the previous three years can be found on the Blackboard page of the course, including solutions and some remarks about how they are affected by changes of the syllabus.

Good luck!