Advanced Quantum Theory

Problem sheet 4
Perturbation theory and the anharmonic oscillator

Set: 3a+b, 5a+b
Due: Monday 27 February 4pm (Solutions can be handed in during the lecture, or in the box in the Maths building, or by email to sebastian.muller@bristol.ac.uk)

1. *Integral kernel*

   In the lecture we looked for the kernel $G(t', t'')$ subject to the conditions

   $$\left(-\frac{m}{\hbar}\right)\left(\frac{\partial^2}{\partial t'^2}+\omega^2\right)G(t',t'')=\delta(t'-t'')$$

   and

   $$G(0,t'')=G(t,t'')=0.$$

   We gave a formula for this kernel and proved it. Now imagine that you don't know this result and determine $G(t', t'')$ from the above formulas by calculation. Consider

   (a) the case $\omega = 0$ corresponding to a free particle, and

   (b) the case of arbitrary $\omega \neq 0$.

2. *Propagator for the anharmonic oscillator*

   Evaluate the integral $\int_0^t G(t', t')^2 dt'$ for the anharmonic oscillator omitted in the lecture. Use this result to determine the propagator $K(0, 0, t)$ ignoring terms of order ϵ^2 and beyond.

3. *Ground state of the anharmonic oscillator*

   We want to determine the ϵ^2 correction to the ground state energy of the anharmonic oscillator.

   (a) First evaluate $G(\frac{\hbar}{i}\beta', \frac{\hbar}{i}\beta'')$ for general β', β'' between 0 and β, assuming that β is large. (It is ok if your approximation breaks down for the case that one of the variables is very close to 0 or β.)

   (b) Use the result from (a) determine the contribution to $\langle 0|e^{-\beta\hat{H}_{\mathrm{anh}}}|0\rangle$ from the Feynman diagram with multiplicity 24.

   (c) Evaluate the remaining diagrams and thus determine the ϵ^2 contribution to the ground state energy.

4. *General initial and final positions*

   In the lecture we have used perturbation theory to determine the propagator of the anharmonic oscillator for $x_f = x_0 = 0$. Explain how one would obtain the propagator for general x_f and x_0. It is sufficient to explain the method and what changes for general initial and final conditions, you don't need to carry through the calculations.

5. *Feynman diagrams*

Use perturbation theory to evaluate the following expressions, and draw the corresponding Feynman diagrams. Here the meaning of $\langle \ldots \rangle$ is the same as in the lecture about the anharmonic oscillator. Your result should involve integrals over products of factors $iG(t', t'')$, and you don't need to evaluate these integrals explicitly.

(a) $\left\langle \exp\left(-\frac{i\epsilon}{\hbar}\int_0^t dt' x(t')^3\right)\right\rangle$ neglecting terms of order ϵ^4 and higher

(b) $\left\langle x(t_1)x(t_2)\exp\left(-\frac{i\epsilon}{\hbar}\int_0^t dt' x(t')^6\right)\right\rangle$ neglecting terms of order ϵ^2 and higher

(c) $\left\langle x(t_1)x(t_2)\exp\left(-\frac{i\epsilon}{\hbar}\int_0^t dt' x(t')^4\right)\right\rangle$ neglecting terms of order ϵ^2 and higher

(d) $\left\langle x(t_1)x(t_2)\exp\left(-\frac{i\epsilon}{\hbar}\int_0^t dt' x(t')^3\right)\right\rangle$ neglecting terms of order ϵ^3 and higher

Drop in session: Thu 10:30-11:30, Fri 10:30-11:30 (Howard House 4.08)