

# Advanced Quantum Theory

## Problem sheet 4 Perturbation theory and the anharmonic oscillator

Please hand in questions 3a+b and 5a+b by Thursday 1 March 4pm. Solutions can be handed in during the lecture, or in the box in the Maths building, or by email to [sebastian.muller@bristol.ac.uk](mailto:sebastian.muller@bristol.ac.uk)

### 1. *Integral kernel*

In the lecture we looked for the kernel  $G(t', t'')$  subject to the conditions

$$\left(-\frac{m}{\hbar}\right) \left(\frac{\partial^2}{\partial t'^2} + \omega^2\right) G(t', t'') = \delta(t' - t'')$$

and

$$G(0, t'') = G(t, t'') = 0.$$

We gave a formula for this kernel and proved it. Now imagine that you don't know this result and determine  $G(t', t'')$  from the above formulas by calculation. Consider

- (a) the case  $\omega = 0$  corresponding to a free particle, and
- (b) the case of arbitrary  $\omega \neq 0$ .

### 2. *Propagator for the anharmonic oscillator*

Evaluate the integral  $\int_0^t G(t', t')^2 dt'$  for the anharmonic oscillator omitted in the lecture. Use this result to determine the propagator  $K(0, 0, t)$  ignoring terms of order  $\epsilon^2$  and beyond.

### 3. *Ground state of the anharmonic oscillator*

We want to determine the  $\epsilon^2$  correction to the ground state energy of the anharmonic oscillator.

- (a) First evaluate  $G(\frac{\hbar}{i}\beta', \frac{\hbar}{i}\beta'')$  for general  $\beta', \beta''$  between 0 and  $\beta$ , assuming that  $\beta$  is large. (It is ok if your approximation breaks down for the case that one of the variables is very close to 0 or  $\beta$ .)
- (b) Use the result from (a) determine the contribution to  $\langle 0|e^{-\beta\hat{H}_{\text{anh}}}|0\rangle$  from the Feynman diagram with multiplicity 24.
- (c) Evaluate the remaining diagrams and thus determine the  $\epsilon^2$  contribution to the ground state energy.

### 4. *General initial and final positions*

In the lecture we have used perturbation theory to determine the propagator of the anharmonic oscillator for  $x_f = x_0 = 0$ . Explain how one would obtain the propagator for general  $x_f$  and  $x_0$ . It is sufficient to explain the method and what changes for general initial and final conditions, you don't need to carry through the calculations.

### 5. Feynman diagrams

Use perturbation theory to evaluate the following expressions, and draw the corresponding Feynman diagrams. Here the meaning of  $\langle \dots \rangle$  is the same as in the lecture about the anharmonic oscillator. Your result should involve integrals over products of factors  $iG(t', t'')$ , and you don't need to evaluate these integrals explicitly.

- (a)  $\left\langle \exp \left( -\frac{i\epsilon}{\hbar} \int_0^t dt' x(t')^3 \right) \right\rangle$  neglecting terms of order  $\epsilon^4$  and higher
- (b)  $\left\langle x(t_1)x(t_2) \exp \left( -\frac{i\epsilon}{\hbar} \int_0^t dt' x(t')^6 \right) \right\rangle$  neglecting terms of order  $\epsilon^2$  and higher
- (c)  $\left\langle x(t_1)x(t_2) \exp \left( -\frac{i\epsilon}{\hbar} \int_0^t dt' x(t')^4 \right) \right\rangle$  neglecting terms of order  $\epsilon^2$  and higher
- (d)  $\left\langle x(t_1)x(t_2) \exp \left( -\frac{i\epsilon}{\hbar} \int_0^t dt' x(t')^3 \right) \right\rangle$  neglecting terms of order  $\epsilon^3$  and higher

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